Parallel Time-Periodic Finite-Element Method for Steady-State Analysis of Rotating Machines

Yasuhito Takahashi[1], Takeshi Iwashita[2], Hiroshi Nakashima[2], Tadashi Tokumasu[3], Masafumi Fujita[3], Shinji Wakao[4], Koji Fujiwara[1], and Yoshiyuki Ishihara[1]
[1] Doshisha University, 1-3 Tatara Miyakodani, Kyotanabe, Kyoto 610-0321, Japan
[2] Kyoto University, Yoshida-Honmachi, Sakyo-ku, Kyoto 606-8501, Japan
[3] Toshiba Corporation Power Systems Company, 2-4 Suehiro-cho, Tsurumi-ku, Yokohama 230-0045, Japan
[4] Waseda University, 3-4-1 Ohkubo, Shinjuku-ku, Tokyo 169-8555, Japan
ytakahashi@mail.doshisha.ac.jp

Abstract — This paper investigates the parallelization of the time-periodic finite-element method in nonlinear magnetic field analyses of rotating machines. The developed method, which can obtain the steady state solutions directly, provides large granularity even in the small-scale problems compared with the ordinary parallel FEM based on the domain decomposition method. Numerical results verify the effectiveness of the developed method.

I. INTRODUCTION

Transient eddy-current analyses taking into account nonlinear magnetic properties frequently require an extremely large number of time steps to obtain steady state solutions. To accelerate the convergence to a steady state, the authors have proposed time-periodic explicit error correction (TP-EEC) method and clarified its effectiveness in analyzing practical electric machines [1].

In this paper, as a different approach to further speed up the steady state analysis of electric machines, we investigate the parallelization of the time-periodic finite-element method (TPFEM), which can obtain steady state solutions directly [2][3], with MPI. The parallel TPFEM can be regarded as the parallelization of time-axis direction in transient analyses. The ordinary parallel finite-element method (FEM) based on the domain decomposition [4]-[6] is generally not effective for small-scale analyses. In contrast, the parallel TPFEM can provide large granularity in parallel computing even in the 2-D problems by treating all nonlinear systems of equations at every time step for a period simultaneously. In addition, the parallel TPFEM is suitable for the analysis which needs a large number of time steps such as motors driven by PWM inverters. Numerical results that verify the effectiveness of the developed method are presented.

II. METHOD OF ANALYSIS

A. Formulation of Time-Periodic Finite-Element Method

A nonlinear system of equations derived from the A-ϕ formulation in a quasi-static field is given by

$$S(\boldsymbol{x})+C\frac{\partial}{\partial t}\boldsymbol{x}=\boldsymbol{f}\,, \qquad (1)$$

where $\boldsymbol{x}$ is the unknown vector, and $\boldsymbol{f}$ is the right-hand-side vector. $S(\boldsymbol{x})$ is generally nonlinear with respect to $\boldsymbol{x}$ because of nonlinear magnetic properties and C is constant. Here, one or half period is divided into n time steps and the θ method is adopted for the time integration scheme. The linearized equations of the TPFEM are expressed as

$$\begin{bmatrix} \tilde{T}_1 & O & \cdots & O & -\tilde{C}_n \\ -\tilde{C}_1 & \tilde{T}_2 & \cdots & O & O \\ O & -\tilde{C}_2 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & O \\ O & O & \cdots & -\tilde{C}_{n-1} & \tilde{T}_n \end{bmatrix} \begin{Bmatrix} \Delta\boldsymbol{x}_1 \\ \vdots \\ \Delta\boldsymbol{x}_n \end{Bmatrix} = \begin{Bmatrix} -\boldsymbol{G}_1 \\ \vdots \\ -\boldsymbol{G}_n \end{Bmatrix}, \qquad (2)$$

$$\tilde{T}_i=\theta S_i+\frac{C}{\Delta t},\ \tilde{C}_i=-(1-\theta)S_i+\frac{C}{\Delta t},\ S_i=\frac{\partial S}{\partial \boldsymbol{x}}(\boldsymbol{x}_i), \qquad (3)$$

where $\Delta\boldsymbol{x}_i$ is the increment of $\boldsymbol{x}_i$, $\boldsymbol{G}_i$ is the residual, and the subscript indicates the time step.

B. Parallelization of TPFEM

The coefficient matrix derived from the TPFEM in (2) is nearly block diagonal. To make the best use of the distinctive structure, we assign the unknowns to each process based on time steps. Fig. 1 shows the example when the number of processes is 3 and n = 9. Inter-process communication is performed only between processes handling previous and next time steps. Therefore, the communication data size per process is constant without depending on the number of processes. Because the input mesh data is the same in all the processes and each process outputs torque or eddy-current loss at assigned time steps independently, input and output can be parallelized easily.

For solving the nonsymmetric linear system (2), we adopt the BiCGstab2 method [7] and the additive Schwarz type ILU preconditioning, in which the ILU preconditioner is used as a local solver in each subdomain. In Fig. 1, for instance, shaded submatrices are ignored in the ILU preconditioning for parallel processing. The convergence rate of the additive Schwarz type preconditioning generally deteriorates as the number of the processes increases because of the ignored submatrices. In the case of the parallel TPFEM, however, it is expected that the preconditioning effect does not deteriorate seriously due to the nearly block diagonal structure of the coefficient matrix.

III. NUMERICAL EXAMPLES

Fig. 2 shows the analyzed induction motor. The number of elements is 13,198. One period is divided into 256 time steps and the slip is set to 1. The number of unknowns for the parallel TPFEM is 3,252,480. All the computations were performed on T2K Open Supercomputer HX600 [8], in which a node consists of four AMD Opteron 8356

$$
\begin{bmatrix}
\tilde{T}_1 & O & O & O & O & O & O & O & -\tilde{C}_9 \\
-\tilde{C}_1 & \tilde{T}_2 & O & O & O & O & O & O & O \\
O & -\tilde{C}_2 & \tilde{T}_3 & O & O & O & O & O & O \\
O & O & -\tilde{C}_3 & \tilde{T}_4 & O & O & O & O & O \\
O & O & O & -\tilde{C}_4 & \tilde{T}_5 & O & O & O & O \\
O & O & O & O & -\tilde{C}_5 & \tilde{T}_6 & O & O & O \\
O & O & O & O & O & -\tilde{C}_6 & \tilde{T}_7 & O & O \\
O & O & O & O & O & O & -\tilde{C}_7 & \tilde{T}_8 & O \\
O & O & O & O & O & O & O & -\tilde{C}_8 & \tilde{T}_9
\end{bmatrix}
\begin{Bmatrix}
\Delta \boldsymbol{x}_1 \\ \Delta \boldsymbol{x}_2 \\ \Delta \boldsymbol{x}_3 \\ \Delta \boldsymbol{x}_4 \\ \Delta \boldsymbol{x}_5 \\ \Delta \boldsymbol{x}_6 \\ \Delta \boldsymbol{x}_7 \\ \Delta \boldsymbol{x}_8 \\ \Delta \boldsymbol{x}_9
\end{Bmatrix}
=
\begin{Bmatrix}
\boldsymbol{G}_1 \\ \boldsymbol{G}_2 \\ \boldsymbol{G}_3 \\ \boldsymbol{G}_4 \\ \boldsymbol{G}_5 \\ \boldsymbol{G}_6 \\ \boldsymbol{G}_7 \\ \boldsymbol{G}_8 \\ \boldsymbol{G}_9
\end{Bmatrix}
\begin{array}{l}
\\ \text{process1} \\ \\ \\ \text{process2} \\ \\ \\ \text{process3} \\ \\
\end{array}
$$

Fig. 1. Parallelization of TPFEM.

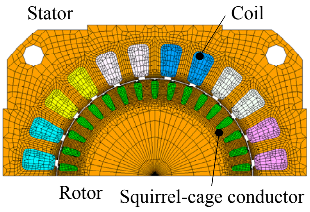

Fig. 2. 2-dimensional meshes of induction motor.

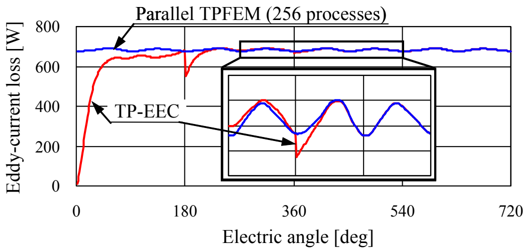

Fig. 3. Eddy-current loss in a squirrel cage conductor.

TABLE I Performance of parallel TPFEM in induction motor model.

Number of processes	Number of total NR iterations	Number of total BiCGstab2 iterations	Total calculation time [sec] (parallel speedup)
1	10	5482	28041.9 (1.00)
8	10	5406	7021.1 (3.99)
16	10	5101	3133.6 (8.95)
32	10	5253	1612.8 (17.39)
64	10	5178	816.9 (34.33)
128	10	5344	448.4 (62.54)
256	10	5738	251.9 (111.32)

processors. Fig. 3 shows the time variation of the eddy-current loss in the squirrel-cage conductor. The numerical results obtained from the parallel TPFEM are in good agreement with those obtained from the sequential transient analysis with the TP-EEC method after 3 corrections. Table I shows the performance of the parallel TPFEM, which indicates the good scalability even in the 2-D problem. Because the convergence rate of the BiCGstab2 method does not deteriorate seriously due to the nearly block diagonal structure of the coefficient matrix, the total number of BiCGstab2 iterations is nearly-unchanged. On the other hand, the calculation time of the sequential transient analysis with the TP-EEC method is 7,049 sec. From the above results, the effectiveness of the parallel TPFEM can be confirmed. The parallel speed-up ratio is about half of the number of processes as shown in Table I. This is because sparse matrix-vector multiplications and forward and backward substitutions, which constitute the main kernels of the iterative solver, are prominently affected by memory throughput compared with the performance of processing cores [9].

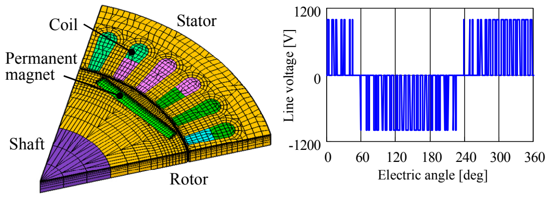

Fig. 4. Mesh of IPM motor and applied voltage waveform.

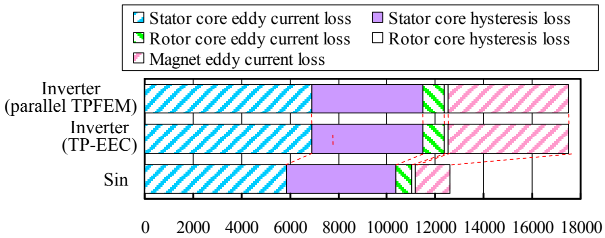

Fig. 5. Comparison of calculated total loss.

As a practical application, we analyze the interior permanent magnet (IPM) motor driven by the PWM inverter whose dc voltage and carrier frequency are 1000 V and 1.5 KHz. Fig. 4 shows the analyzed mesh and applied voltage waveform. The amplitude and phase of the armature current are set to 750 A and 75 deg. The number of elements is 27,680 and one period is divided into 1,024 time steps. The number of unknowns is 40,850,432. The calculation time with 256 MPI processes is 8,979 sec. Fig. 5 shows calculated total loss. The eddy-current loss in the magnet increases significantly due to the carrier harmonics compared with the loss when applying sinusoidal voltage. The detail of the parallel TPFEM and more numerical results will be included in the full paper.

IV. REFERENCES

[1] Y. Takahashi, T. Tokumasu, A. Kameari, H. Kaimori, M. Fujita, T. Iwashita, and S. Wakao, "Convergence Acceleration of Time-Periodic Electromagnetic Field Analysis by Singularity Decomposition-Explicit Error Correction Method," *IEEE Trans. Magn.*, vol. 46, no. 8, pp. 2947-2950 (2010).

[2] T. Hara, T. Naito, and J. Umoto, "Time-periodic finite element method for nonlinear diffusion equations," *IEEE Trans. Magn.*, vol. 21, no. 6, pp. 2261-2264 (1985)

[3] T. Nakata, N. Takahashi, K. Fujiwara, K. Muramatsu, H. Ohashi, and H. L. Zhu, "Practical Analysis of 3-D Dynamic Nonlinear Magnetic Field Using Time-Periodic Finite Element Method," *IEEE Trans. Magn.*, vol. 31, no. 3, pp. 1416-1419 (1995)

[4] Y. Saad, "Iterative Methods for Sparse Linear Systems," Second ed., SIAM, Philadelphia, PA (2003).

[5] T. Nakano, Y. Kawase, T. Yamaguchi, and M Nakamura, "Parallel Computing of Magnetic Field for Rotating Machines on the Earth Simulator," *IEEE Trans. Magn.*, vol. 46, no. 8, pp. 3273-3276 (2010)

[6] A. Takei, S. Sugimoto, M. Ogino, S. Yoshimura, and H. Kanayama, "Full Wave Analyses of Electromagnetic Fields With an Iterative Domain Decomposition Method," *IEEE Trans. Magn.*, vol. 46, no. 8, pp. 2860-2861 (2010)

[7] M. H. Gutknecht, "Variants of Bi-CGSTAB for Matrices with Complex Spectrum," *SIAM J. Sci. Comput.*, vol. 14, no. 5, pp. 1020-1033, 1993.

[8] H. Nakashima "T2K Open Supercomputer: Inter University and Inter-Disciplinary Collaboration on the New Generation Supercomputer," *Proc. Intl. Conf. Informatics Education and Research for Knowledge-Circulating Society*, pp.137–142 (2008).

[9] S. Williams, L. Oliker, R. Vuduc, J. Shalf, K. Yelick, J. Demmel, "Optimization of Sparse Matrix-Vector Multiplication on Emerging Multicore Platforms," *Proc. SC07* , no. 18 (2009).